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**DIMENSIONLESS CHARACTERISTICS OF
TURBULENCE IN THE ATMOSPHERIC LAYER
NEAR THE GROUND**

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A universal law of velocity and temperature distribution is assumed to exist from dimensional considerations for regions near a smooth ground surface, neglecting Coriolis forces and molecular viscosity and thermal conductivity variations. Plots of data confirm this.

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Temperature nonuniformity of the atmosphere has an important effect on the characteristics of turbulent transfer. An attempt was made in the papers of A. M. Obukhov [1] and A. S. Monin [2] to analyze the effect of stratification on the turbulence conditions, based on methods of similarity theory. In this Note methods of similarity theory are applied to the analysis of empirical wind and temperature distribution data in the atmospheric layer near the earth.

In analyzing turbulent conditions in the air layer near the earth over a uniform surface the effect of Coriolis forces can be neglected, along with the influence of molecular viscosity and thermal conductivity, while taking into account small density variations caused by temperature but not by pressure variations. The temperature variations will be considered small in comparison with the average absolute temperature of the medium T_0 . With these simplifications the basic equations contain only one dimensional constant, g/T_0 (g is the acceleration of gravity), representing the effect of buoyant forces.

The boundary layer can be characterized by the condition of constant turbulent friction (independence of z)

$$\tau = -\rho \overline{u'w'} = \text{const} \quad (1)$$

and the condition of constant turbulent heat flux

$$q = c_p \rho \overline{T'w'} = \text{const}, \quad (2)$$

where ρ is the air density; c_p is the specific heat capacity; and u' , w' , and T' are the fluctuations of the horizontal and vertical components of wind velocity, and the temperature, respectively. Conditions (1) and (2) can be obtained from the averaged hydrodynamic equations under the assumption of quasi-steady-state conditions. From the constancy of τ and q it is natural to take them as basic external parameters which determine the turbulent conditions in each concrete case. It is convenient to replace τ and q by the equivalent quantities:

$$v_* = \sqrt{\frac{\tau}{\rho}} = \text{const}, \quad (3)$$

$$\frac{q}{c_p \rho} = \text{const}. \quad (4)$$

Forming the characteristic scale quantities of velocity, temperature, and altitude from the external parameters v_* , $q/c_p\rho$, and the basic dimensional constant g/T_0 , we can represent the average flow velocity $v(z)$ and the temperature $T(z)$ as functions of altitude in the form

$$v(z_2) - v(z_1) = \frac{v_*}{L} \left[f_1\left(\frac{z_2}{L}\right) - f_1\left(\frac{z_1}{L}\right) \right], \quad (5)$$

$$T(z_2) - T(z_1) = -\frac{q}{\kappa v_* c_p \rho} \left[f_2\left(\frac{z_2}{L}\right) - f_2\left(\frac{z_1}{L}\right) \right], \quad (6)$$

where z_2 and z_1 are the altitudes of two arbitrary layers lying above the zone of roughness ($z_2 > z_1 \gg h_0$)¹, and L is the characteristic altitude scale quantity, determined uniquely from considerations of dimensionality:

$$L = -\frac{v_*^3}{\kappa \frac{q}{T_0} \frac{g}{c_p \rho}}. \quad (7)$$

In formulas (5), (6), and (7) the numerical von Karman constant has been included for convenience. A value of $L > 0$ corresponds to conditions of stable stratification (for $q < 0$ the heat flow is directed downward), and $L < 0$ ² corresponds to unstable stratification ($q > 0$). The momentum transfer coefficient κ and the coefficient of turbulent thermal conductivity K_T can be determined formally, setting:

$$K = \frac{\tau}{\rho \frac{dv}{dz}} = \frac{\kappa v_* L}{f_1(z/L)},$$

$$K_T = -\frac{q}{c_p \rho \frac{dT}{dz}} = \frac{\kappa v_* L}{f_2(z/L)}.$$

We assume that the turbulent Prandtl number $K_T \cdot K = \alpha = \text{const}$. In practice α is close to unity, and we can set approximately $K_T = K$ ³. It follows that $f_1'(\zeta) = f_2'(\zeta)$ and $f_1(\zeta) = f_2(\zeta)$, i.e., we are dealing with a single universal function determinable from the sum of the observations carried out under different conditions. Certain properties of this function can be derived from general considerations. For $q \rightarrow 0$ ($L \rightarrow \infty$) we are close to conditions of neutral stratification, for which the well known logarithmic law of von Karman applies:

$$v(z_2) - v(z_1) = \frac{v_*}{\kappa} \ln \frac{z_2}{z_1}; \quad (8)$$

¹ It is convenient to use velocity and temperature differences rather than the values of v and T themselves, since the latter must contain the dependence on conditions at the surface (e.g. roughness and thermal contact characteristics) in explicit form. The "Universal" functions $f_1(\zeta)$ and $f_2(\zeta)$ ($\zeta = z/L$) are determined accurately up to an additive constant.

² [Translator's note: lit., " $L > 0$ ".]

³ The case $\alpha \neq 1$ can be converted to that discussed by changing the temperature scale.

whence it follows that for $\zeta \rightarrow 0$, $f(\zeta) = \ln \zeta + O(\zeta)$. Assuming that the expansion for $f(\zeta)$ can be $f(\zeta) = \ln \zeta$ for $|z/L| < 1$, we obtain the approximations:

$$v(z_2) - v(z_1) \approx \frac{v_*}{\alpha} \left(\ln \frac{z_2}{z_1} + \beta \frac{z_2 - z_1}{L} \right), \quad (9)$$

$$T(z_2) - T(z_1) \approx - \frac{q}{\alpha v_* c_p \rho} \left(\ln \frac{z_2}{z_1} + \beta \frac{z_2 - z_1}{L} \right). \quad (10)$$

The coefficients in the formulas (9) and (10) can be determined from empirical data, approximating the observed data on wind and temperature by formulas of the form:

$$v(z) = v_1 \ln \frac{z}{H} + v_2 \frac{z - H}{H} + v(H), \quad (11)$$

$$T(z) = T_1 \ln \frac{z}{H} + T_2 \frac{z - H}{H} + T(H), \quad (12)$$

where H is a certain average observation altitude. Processing of the data from observations of two expeditions of the Main Geophysical Observatory [3,4] gave values for the numerical constant $\beta \approx 0.6$ which were in close agreement. Finding v_1 and T_1 from observations in the lower part of the layer near the earth, it was possible to calculate the basic characteristics v_* , q , and the scale L in each individual case:

$$v_* = \alpha v_1, \quad q = -\alpha^2 c_p \rho v_1 T_1, \quad L = - \frac{v_1^2}{\frac{\kappa}{T_0} T_1}.$$

We processed data on the gradient measurements (of wind and temperature) obtained in expeditions of the Main Geophysical Observatory in 1945, 1947, and 1950 [3-5] and in the 1951 expedition of the Geophysical Institute of the AN SSSR, with the aim of determining the universal function $f(\zeta)$. In each individual case the quantities v_1 and T_1 (from measurements in the bottom 4 m) were determined with the aid of an approximation of the profiles in the bottom 4 m by formulas of the type of (9) and (10), with $\beta = 0.6$, with simultaneous calculation of L . Following this, all the data on wind distribution were divided into two groups -- the cases with stable and unstable stratification ($L > 0$ and $L < 0$) -- and plotted on a common graph in special dimensionless coordinates. As the "initial altitude" we took $z_1 = |L|/2$. The dimensionless velocity differences $\frac{v_*}{v_*} [v(z_2) - v(z_1)]$ were plotted as functions of the dimensionless altitude.

The right part of the graph (see Fig. 1) corresponds to the function $f(\zeta) = f(1/2)$ (stable stratification, $\zeta > 0$), and the left part ($\zeta < 0$) corresponds to the function $f(\zeta) = f(-1/2)$. Bearing in mind the limited accuracy of the observations and the fact that the data from four expeditions, gathered under completely different conditions, have been generalized in one plot, the agreement with the hypothesis of the existence of the universal function (5) can be considered completely satisfactory.

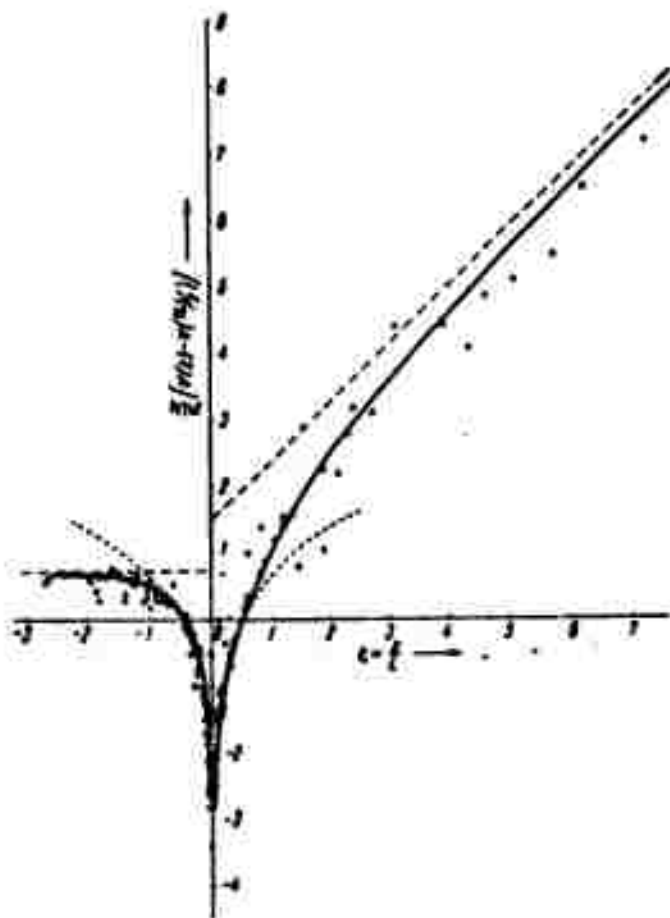


Fig. 1.

Fig. 1.

The empirical universal function of wind distribution with height already displays a pattern of deviation from the logarithmic law (the latter shown as a dotted line) at $z > |L|/2$. At very high instability the curve has a tendency to approach a constant, and at high stability it can be approximated by a straight line. The asymptotic behavior of $f(\zeta)$ in the limiting cases of $|\zeta| \gg 1$ was studied theoretically by Obukhov [1] and Monin [2] and agrees with empirical data shown in Fig. 1. Under strong thermal instability ($\zeta \rightarrow -\infty$) the transfer coefficient in the limiting case is determined only by the thermal factor and is independent of v_* ⁴. In this case an expression for K is obtained from dimensional considerations:

$$K(z) \sim c \left(\frac{\alpha}{T_0} \frac{g}{c_p} \right)^{1/4} z$$

⁴ At $\zeta \rightarrow -\infty$ we approach conditions of free convection, which is also possible in the absence of a gradient in the average velocity ($v_* = 0$). The case of self-similar free convection was studied by Ya. V. Zel'dovich [6].

(c is a numerical constant), whence the asymptotic form is obtained for the universal function:

$$f(\zeta) \approx f(-\infty) - c_1 |\zeta|^{-1/2}, \quad c_1 = \frac{3}{2}.$$

In the case of stable stratification the Richardson number

$$Ri = \frac{g}{T_0} \frac{dT/ds}{(dv/ds)^2} = f'(\zeta) > 0$$

increases with altitude, but cannot exceed a certain constant (for $Ri > Ri_{cr}$ turbulence is in general impossible).

Consequently, at $\zeta \rightarrow \infty$, $Ri(\zeta) \rightarrow c_2$ and

$$f(\zeta) \approx c_2 \zeta + \text{const.}$$

The use of the scale characteristic L (the altitude of the layer of dynamic turbulence) and universal functions may be of practical interest for correlation of gradient measurements in microclimatic studies.

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